WOCOMAL

Varsity Meet #4

April 2, 2003

Bartlett High School
Webster, Massachusetts

WoCoMaL

April 2, 2003 Varsity Meet#4

ROUND#1: **Elementary Number Theory << No Calculators >>**

1. A palindrome is a number which reads the same when its digits are written in reverse order. A car's odometer reads 15,951 miles. Find the least number of miles required for the next palindrome to appear.

2. In the year 2427 of our calendar, a wormhole will appear to the LØTSA Galaxy, inhabited by LØTSAns, who use the LØTSAdecimal number system. The digits L, Ø, T, S, and A (in that order) are used to represent the digits 10 thru 14 of their base 15 system. Convert 2427 from our base ten into its LØTSAdecimal equivalent.

3. The number 1000 can be written as the sum of *N* consecutive positive integers. Find all possible *N*.

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Bromfield, Tahanto, Hudson & QSC

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ROUND#2: **Algebra 1**

1. For what positive value of x is $x+2$ the reciprocal of $x-2$?

2. Points $A=(1,2)$, $T=(5,-3)$, and $E=(-3,7)$ are collinear. Compute the y-intercept of the line that contains them.

3. Pam is five years older than Louie. In six years, Pam will be twice as old as Louie was four years ago. How old will Louie be when Pam is twice as old as she is now?

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Burncoat, Auburn, Algonquin

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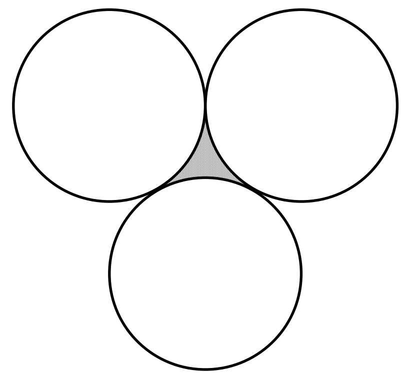
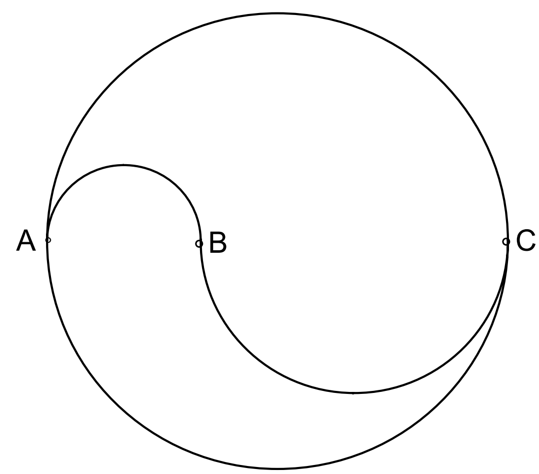
ROUND#3: **Geometry** **<< Answers exact or rounded to three decimal places. >>**

1. What is the sum of the numbers of the faces, edges, and vertices of a cube?

2. Three congruent circles, externally tangent in pairs, each have radius 6. Compute the area of the curvilinear triangle between them.



3. Two semicircles joining A and C, a smaller one from B to C, and an even smaller one from A to B divide the large circle into two regions. If $AB = 2x$ and $BC = 2y$, in terms of x and y, find the ratio of the areas of the regions. Simplify.



Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Burncoat, Bancroft, MT(1/03)

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ROUND#4: **Logs, Exponents & Radicals** **<< No Calculators >>**

1. If $\log_N 64 + \log_N 16 = \log_N x$, then $x = ?$

2. If $A\log_{1000} 5 + B\log_{1000} 2 = C$, express $A + B$ in terms of C.

3. The unique solution to $\log_x 876 = 2 \cdot \log_{x+5} 876$ is of the form $\dfrac{1+\sqrt{a}}{c}$.
   Find a .

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Assabet Valley, Auburn, Doherty

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ROUND#5: **Trigonometry**

1. A 98 foot extension ladder rests on top of a ladder truck with its base 11 feet above the ground. When the angle of elevation of the ladder is 73°, how high up the building will it reach? [Answer to nearest foot.]

2. Solve for θ where $0^\circ \le \theta < 360^\circ$:

$$1 - \cos\theta = \sqrt{3}\sin\theta$$

3. Ranger Rick and Friend are stationed exactly 20 kilometers apart at two lookouts on a north-south forest road. Rick spots a fire at 37° west of south, and Friend sees the same fire at 57° west of north. To the nearest hundredth of a kilometer, how far is the fire from the forest road?

Answer here: 1. (1 pt.) __________

2. (2 pts.) _________

3. (3 pts.) _________

Bartlett, St. John's, Bromfield

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Team Round: **<< Answers exact or rounded to three decimal places. >>**

1. Solve for X: $\dfrac{11_{(base\ 2)}}{21_{(base\ 3)}} = \dfrac{33_{(base\ 4)}}{X_{(base\ 6)}}$

2. If $x^2 + y^2 = 58$ and $xy = 21$, find the largest possible value of $x + \dfrac{1}{y}$.

3. An isosceles trapezoid has its upper base equal in length to the two legs. The altitude of the trapezoid is 4 and the lower base is 11. How long is the upper base?

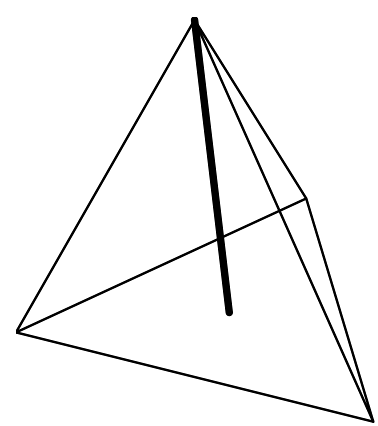
4. Solve $\sqrt{4x+1} - \sqrt{2x+1} = 2$ for x.

5. If $\log_{\sin x}(\cos x) = \frac{1}{2}$, compute the value of $\csc x$ in simplified radical form.

6. If A, B, C are digits in this subtraction problem, compute $A + B + C$.

$$\begin{array}{r} 7\ A\ 2 \\ -\ 4\ 8\ B \\ \hline C\ 7\ 3 \end{array}$$

7. Compute the altitude of a regular tetrahedron (4 equilateral triangle faces) having edges of length 18.



8. If $\cos\theta = \dfrac{1}{9}$ and $0 < \theta < 2\pi$, compute the value of $\sin\left(\frac{1}{2}\theta\right)$.

9. A circle is inscribed within a regular hexagon having perimeter $12\sqrt{2}$. Compute the area of the region between the circle and the hexagon.

Assabet, Quaboag, Hudson, Algonquin, Worcester Acad., Westboro, QSC, Clinton, St.Peter-Marian

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April 2, 2003 Team Round Varsity Meet#4

2 Points Each

Answers here ↓ :

1.________________________

2.________________________

3.________________________

4.________________________

5.________________________

6.________________________

7.________________________

8.________________________

9.________________________

School:______________________________

Team#: ______

Players' Names ↓ :

#1:______________________

#2:______________________

#3:______________________

#4:______________________

#5:______________________

WOCOMAL Answers Varsity Meet #4 April 2, 2003

R#1: 1. 110 or 110 miles

2. $L\O T$ or $L\O T_{15}$

3. 5 and 25 (need both)

R#2: 1. $\sqrt{5}$ or $x=\sqrt{5}$

2. $\frac{13}{4}=3\frac{1}{4}=3.25$

3. 43

R#3: 1. 26

2. $36\sqrt{3}-18\pi \approx 5.805$

3. Either $\dfrac{x}{y}$ or $\dfrac{y}{x}$

R#4: 1. 1024 or $x=1024$ or 2^{10} or 4^5

2. $6C$ or $A+B=6C$

3. 21 or $a=21$

R#5: 1. 105 or 105 ft [104.72]

2. 0° and 120° [or 0 & $\frac{2\pi}{3}$]

3. 10.12 or 10.12 km or
10 km, 120 m or 10,120 m

Team:

1. 55 or $55_{(base\ 6)}$

2. $\frac{22}{3}=7\frac{1}{3}=7.\overline{3}\approx 7.333$

3. 5

4. 12 or $x=12$

5. $\dfrac{1+\sqrt{5}}{2}\approx 1.618$

6. 17

7. $6\sqrt{6}\approx 14.697$

8. $\dfrac{2}{3}=0.\overline{6}\approx 0.667$

9. $12\sqrt{3}-6\pi \approx 1.935$

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V4 - Solutions Apr. 2, 2003

Round#1 1. $16,061-15,951=110$ *miles*

2. $2427=15\times161+12=15\times(15\times10+11)+12=10\times15^2+11\times15+12=LØT_{15}$

3. 1000 must be the product of N and the average of the numbers in the list, also the middle number of the list. You quickly discover that N cannot be even. Since 1000 contains only factors of 2 and 5, the only remaining viable candidates for N are 5, 25, and 125. 5 works with middle number 200; 25 works with middle number 40, but 125 cannot work with middle number 8.

Round#2 1. The only positive solution of $x+2=\dfrac{1}{x-2}$ is $\sqrt{5}$

2. The y-int of $5x+4y=13$ is $13/4$

3. $P=5+L$ and $P+6=2(L-4)$ implies $P=24$ and $L=19$.
So, age = 19 + 24 = 43

Round#3 1. $F+E+V=6+12+8=26$

2. Connect centers forming an equilateral triangle and three 60° sectors.
So, area = $\dfrac{12^2\sqrt{3}}{4}-3\left(\frac{1}{6}\times\pi\,6^2\right)=36\sqrt{3}-18\pi$

3. $$\frac{smaller}{larger}=\frac{\left[\frac{1}{2}\pi(x+y)^2-\frac{1}{2}\pi y^2\right]+\frac{1}{2}\pi x^2}{\left[\frac{1}{2}\pi(x+y)^2-\frac{1}{2}\pi x^2\right]+\frac{1}{2}\pi y^2}=\frac{\left[(x+y)^2-y^2\right]+x^2}{\left[(x+y)^2-x^2\right]+y^2}=\frac{2x^2+2xy}{2xy+2y^2}=\frac{x}{y}$$

Round#4 1. $x=64\times16=1024$

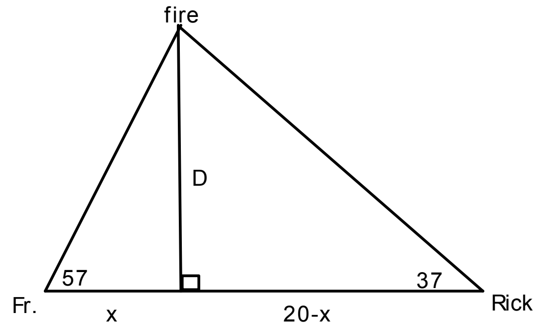
2. $A\log_{1000}5+B\log_{1000}2=\log_{1000}5^A+\log_{1000}2^B=\log_{1000}\left(5^A\cdot2^B\right)=C$ implies $5^A\cdot2^B=1000^C=2^{3C}\cdot5^{3C}$, which implies $A=B=3C$

3. Use the *change of base rule* to write the equation as $\dfrac{\log 876}{\log x}=\dfrac{2\log 876}{\log(x+5)}$.
Cancelling and clearing, we get $\log(x+5)=2\log x=\log x^2$. So, we want x > 0 such that $x+5=x^2$. $x=\dfrac{1+\sqrt{21}}{2}$ and $a=21$

Round#5 1. Height = $11+98\times\sin73^\circ\approx104.72$ *feet*

2. Squaring both sides and replacing $\sin^2\theta$ with $1-\cos^2\theta$, the simplified quadratic in cosine becomes $2\cos^2\theta-\cos\theta-1=0$. It factors and yields solutions $\theta=0^\circ,\ 120^\circ,\ 240^\circ$. But 240° is extraneous

3. $\frac{20-x}{D} = \cot 37^\circ$ and $\frac{x}{D} = \cot 57^\circ$.
Add to obtain $\frac{20}{D} = \cot 37^\circ + \cot 57^\circ$.
So, $D = \frac{20}{\cot 37^\circ + \cot 57^\circ} \approx 10.119 km$



<u>Team</u> 1. Converted to base 10, the proportion becomes $\frac{3}{7} = \frac{15}{X}$. So, $X = 35_{10} = 55_6$

2. Sols for x and y are ± 3 and ± 7. From these, the largest possible $x + \frac{1}{y}$ is $7 + \frac{1}{3}$

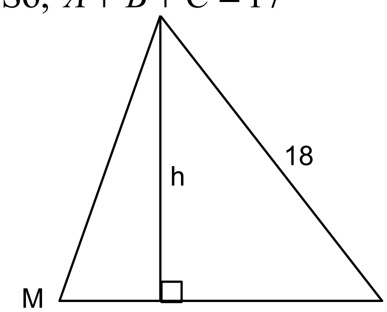
3. Draw two alts, and in one of the rt Δs use the Pythagorean Theorem. Up base = S.
$S^2 = 4^2 + \left(\frac{11-S}{2}\right)^2$ implies $S = 5$

4. Isolate and square ... twice. Sols are $x = 0$ or $x = 12$, but 0 doesn't check.

5. In exponential form, this becomes $cox = \sin^{\frac{1}{2}} x$. Square, replace $\cos^2 x$ to obtain $1 - \sin^2 x = \sin x$. Multiply both sides by $\csc^2 x$ right now. Sols for quadratic in $\csc x$ are $\frac{1 \pm \sqrt{5}}{2}$, and only $\frac{1+\sqrt{5}}{2}$ is positive for log base

6. $B = 9$, $A = 5+1$ is a borrowed from 6, and $C = 2$ because 7 has been borroed from.
So, $A + B + C = 17$

7.



Pass a plane thru an edge and the midpoint M of the opposite edge. This triangle has sides of lengths $9\sqrt{3}$, $9\sqrt{3}$, and 18. The foot of the altitude h divides the median in 1 to 2. So, use $6\sqrt{3}$ with 18 and the Pythagorean Thm., and $h = 6\sqrt{6}$

8. $\sin^2\left(\frac{1}{2}\theta\right) = \frac{1-\cos\theta}{2} = \frac{1-\frac{1}{9}}{2} = \frac{4}{9}$ implies $\sin\left(\frac{1}{2}\theta\right) = \pm\frac{2}{3}$. But $0 < \theta < 2\pi$ means $0 < \frac{\theta}{2} < \pi$, placing θ in either the 1st or 2nd quadrant. So, $\sin\left(\frac{1}{2}\theta\right) = \frac{2}{3}$ only

9. Side of hexagon is $2\sqrt{2}$;
radius of circle is $\frac{2\sqrt{2}}{2} \times \sqrt{3} = \sqrt{6}$
because of 30-60-90 Δ.
Area of hex is $6\left(\frac{(2\sqrt{2})^2\sqrt{3}}{4}\right) = 12\sqrt{3}$;
area of circle is $\pi(\sqrt{6})^2 = 6\pi$.
Answer is $12\sqrt{3} - 6\pi$